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SUBJECT: Analysis of MSRE Zero-Power Flux Noise Using Digital Techniques

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FROM: T. W. Kerlin and S. J. Ball

ABSTRACT

The record of the flux noise obtained during the zero-power operation of the MSRE with fuel circulating was analyzed by two different digital computer techniques. The indirect method consisted of calculating the autocorrelation function of the flux noise and subsequent Fourier analysis of this autocorrelation function to give the power spectral density. The direct method used a digital simulation of a band pass filter to concentrate the signal in the desired frequency range. The output of this filter was then squared and time-averaged to give the power spectral density.

Both methods were found to give comparable results at comparable costs. The results were also found to give reasonable agreement with previously published results obtained with analog methods. The value of β/ℓ obtained by the digital method is 16.2 compared with 14.8 obtained in the earlier, analog study.

CONTENTS

INTRODUCTION

Digital techniques were used to analyze the noise record obtained during the zero-power run of the MSRE. These data were previously analyzed by analog methods by Roux and Fry.[1] The purpose of the present analysis was to supplement the analog results and to further test the digital methods. One of the digital techniques used in this analysis had previously been used successfully in analysis of ORR noise data.[2]

METHODS OF ANALYSIS

Indirect Method

The steps in the indirect method are:

1. Calculate the autocorrelation function, $C_{11}(\tau)$, of the noise record using the following expression:

$$C_{11}(\tau) = \frac{1}{\tau_m} \int_o^{\tau_m} \varphi(t)\ \varphi(t + \tau)\ dt \quad , \tag{1}$$

where

τ_m = maximum correlation time, and

φ = the neutron flux signal.

2. Fourier analyze the autocorrelation function. Since it is an even function with period $2\tau_m$, we obtain:

$$F_k \left\{ C_{11}(\tau) \right\} = \frac{2}{\tau_m} \int_o^{\tau_m} C_{11}(\tau)\ \cos \frac{k\pi}{\tau_m}\ d\tau \quad . \tag{2}$$

3. Apply necessary corrections. These include:

a. Spectral windows to compensate for the fact that the Fourier analysis uses a finite integration time.

b. Filter corrections to remove the effect of a low-pass filter used to eliminate aliasing.

c. Background corrections.

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[1] D. N. Fry and D. P. Roux, "Results of Neutron Fluctuation Measurements Made During the MSRE Zero-Power Experiment," USAEC Report ORNL-CF-65-10-18, October 29, 1965.

[2] Letter from T. W. Kerlin to D. P. Roux, September 17, 1965. Subject: Digital Calculation of the Power Spectral Density from Noise Data.

The corrected Fourier coefficient, $F_k\{C_{11}(\tau)\}$, at the frequency, $k\pi/\tau_m$ radians/sec, is the power spectral density (PSD) at that frequency.

Direct Method

In the direct method, the digitized noize signal is used as the input or forcing function to the differential equations representing a narrow band pass filter, and the resulting output of the filter is squared and integrated. The matrix exponential technique[3] is used to solve for the transient response of the filter, which has the characteristics of a quadratic lag and a transfer function:

$$H(j\omega) = \frac{j\omega}{\omega_o^2 + 2\,\delta\,\omega_o\,j\omega - \omega^2} \quad . \tag{3}$$

The center or resonant frequency of the filter is ω_o, and the band width increases with increasing damping factor δ. The PSD may be computed from

$$\text{PSD} = \frac{\overline{q^2}}{\int_0^\infty \left|H(j\omega)\right|^2 d\omega}\left(\frac{\text{volts}^2}{\text{rad/sec}}\right), \tag{4}$$

(where $\overline{q^2}$ is the mean square filter output) if it is assumed that the PSD is constant within the band pass. For this filter

$$\int_0^\infty \left|H(j\omega)\right|^2 d\omega = \frac{\pi}{4\,\delta\,\omega_o}\ \text{(radians/sec)} \quad .$$

Provisions are also made in the code for correcting the PSD for any low-pass filter that may have been used to prevent aliasing, and for calculating the percent standard deviation of the PSD estimate.

MSRE DATA

The data previously used in the analog analysis[1] were digitized on the Millisadic digitizer. The data included records taken for the

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[3] S. J. Ball and R. K. Adams, "MATEXP, A General Purpose Digital Computer Program for Solving Nonlinear Ordinary Differential Equations by the Matrix Exponential Method," USAEC ORNL Report in preparation.

reactor critical and for the background noise observed when the reactor was shutdown. The case considered was for the reactor primary salt circulating with no bubbles. The noise record for the critical reactor was passed through a low-pass filter consisting of a first order lag with a time constant of 0.0047 sec, then digitized with a sampling interval of 0.00284 sec. The background noise was also filtered and digitized in the same manner. Approximately 36,000 time points were used for both cases.

RESULTS

Indirect Method

Figures 1 through 3 show the autocorrelation functions obtained in the indirect analysis. All calculated points are plotted for the shorter correlation times, but only every tenth point was included after the curve had leveled out at longer correlation times. Figure 1 shows the autocorrelation function for signal plus background. Figure 2 shows the autocorrelation function for background only. The results shown in Fig. 3 were obtained by subtracting the background autocorrelation function from the autocorrelation function for signal plus background. This can be done if the signal and the background are uncorrelated. To show this, take a signal composed of uncorrelated time functions x and y, and calculate the autocorrelation function

$$\begin{aligned} C_{11}(\tau) &= \frac{1}{T}\int_0^T [x(t) + y(t)][x(t+\tau) + y(t+\tau)]\, dt \\ &= \frac{1}{T}\int_0^T x(t)\, x(t+\tau)\, dt + \frac{1}{T}\int_0^T y(t)\, y(t+\tau)\, dt \\ &+ \frac{1}{T}\int_0^T x(t)\, y(t+\tau)\, dt + \frac{1}{T}\int_0^T y(t)\, x(t+\tau)\, dt \quad . \end{aligned} \tag{5}$$

Since x and y are uncorrelated, the last two integrals are zero and

$$C_{11}(\tau) = \frac{1}{T}\int_0^T x(t)\, x(t+\tau)\, dt + \frac{1}{T}\int_0^T y(t)\, y(t+\tau)\, dt \quad . \tag{6}$$

Thus, if x is the signal and y is the background, we see that we get the autocorrelation function of the signal by subtracting the autocorrelation function of the background from the autocorrelation function of the composite signal. The improvement obtained from the background correction is quite apparent if one compares Fig. 1 with Fig. 3.

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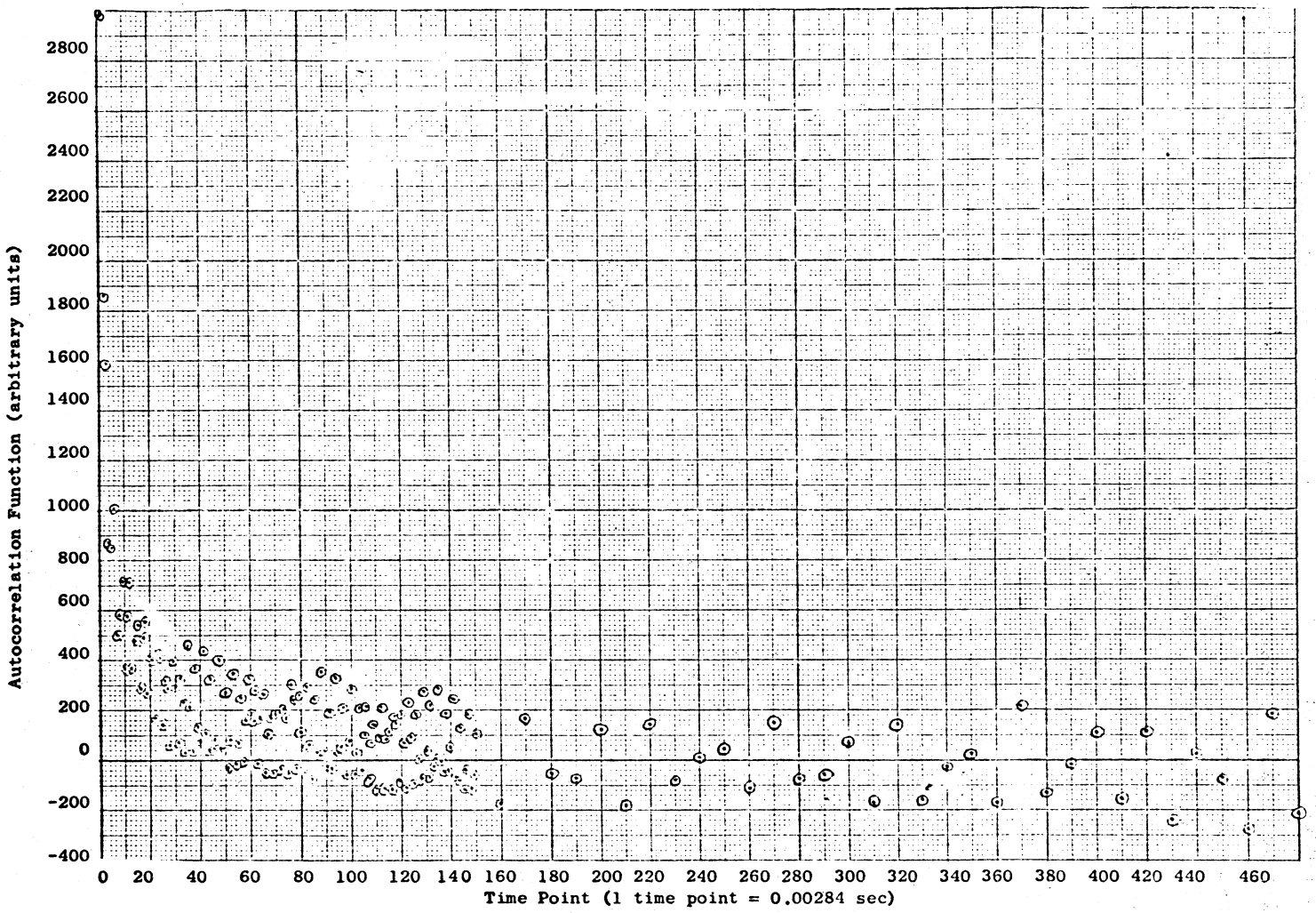

Figure 1. Autocorrelation Function — Signal + Background.

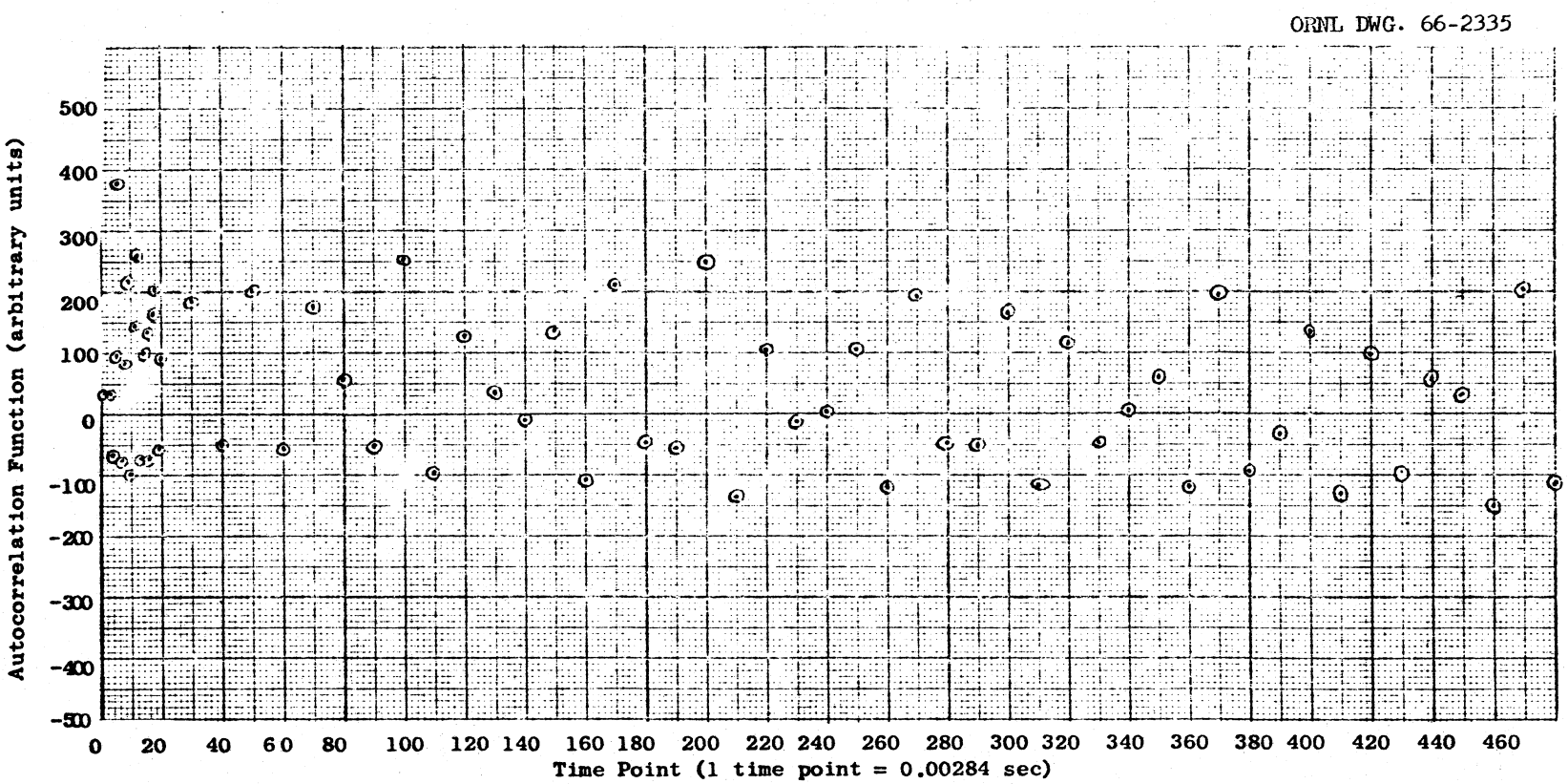

Figure 2. Autocorrelation Function — Background Only.

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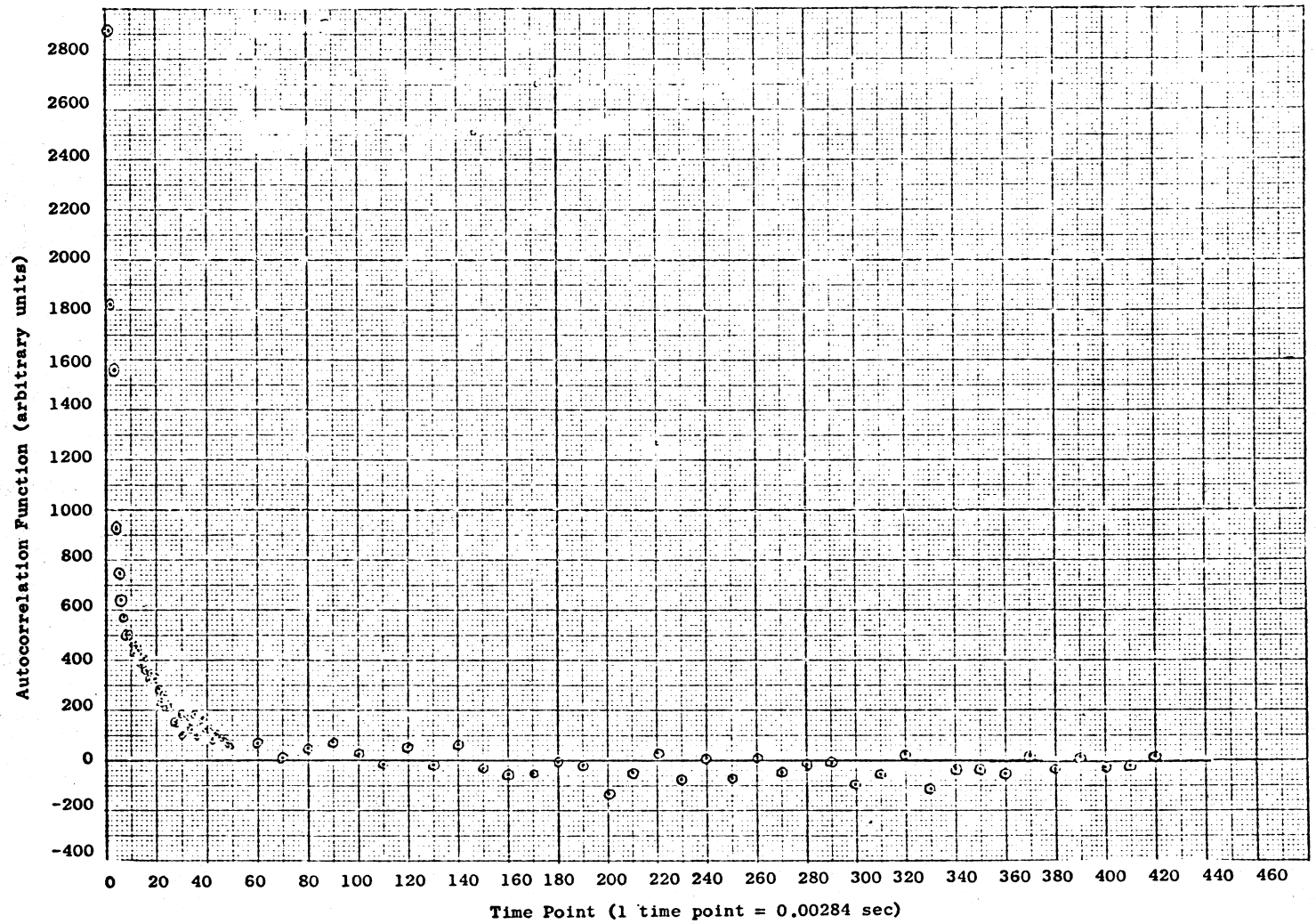

Figure 3. Autocorrelation Function of Signal + Background minus Autocorrelation Function of Background Only.